Mathematic modelling of thermodynamic effects in a gas formation well bore zone

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Abstract

With the use of ideas of stepwise recording of process and medium characteristics, the methodology of mathematical modelling of filtration and mass-exchanging processes in a formation wellbore zone has been suggested. The methodology considers thermodynamic effects which appear as a result of gas throttling through micro fractures of shale rock when, on condition of quasistationary filtration flow, displacement process is described by specially modified Darcy's law with critical gradient.

Model problem of hydrodynamic effects investigation in a formation wellbore zone in case of gas filtration in a formation with low permeability has been formulated. On the example of radial gas flow the peculiarities of temperature field division algorithm building have been illustrated.

Key Words: *Joule-Thomson effect, iteration method, non-linear filtration, sedimentary shale rock.*

Structural features of shale sedimentary rocks, their low permeability and conditions of gas occurrence there demand search of effective technologies of development of such fields, which would take into account regularities of gas drainage from the system of fractures and micro fractures to wellbores. This process as a result of conversion of work performed by gas while transition from fractures into micro fractures and vice versa is accompanied by heat emission and leads to unbalanced gas expansion. Apart from thermodynamic effects, caused by such a motion it is also worth parallel considering of non-linear effects in formation wellbore zones, caused by the fact that pressure gradient exceeds its certain critical value.

In this paper on the basis of idea of stepwise recording of process and medium characteristics, methodology of mathematical modelling of filtration and mass-exchanging processes in a formation wellbore zone has been suggested. The methodology considers thermodynamic effects which appear as a result of gas throttling through micro fractures of shale rock when on condition of quasistationary filtration flow, displacement process is described by specially modified Darcy's law with critical gradient.

Let's analyse a model problem of thermodynamic effects investigation in a formation wellbore zone G_z, which appear as a results of gas throttling through shale rock micro fractures. For mathematical formulation of liquids filtration problem in porous medium let's write down equation of continuity and equation of motion which are as follows [1–6]

$$div\rho\boldsymbol{\upsilon} = 0\,,\ \ \boldsymbol{\upsilon} = -\frac{k\chi(I, I_{cr})}{\mu}\,\boldsymbol{grad}\ p \qquad (1)$$

at correspondent conditions on reservoir boundaries $p\big|_{L_*} = p_*$, $p\big|_{L^*} = p^*$ $(p_* > p^*)$. Here $I = I(x, y) = |\,\boldsymbol{grad}\ p(x, y)| = \sqrt{p_x^2 + p_y^2}$ is the pressure gradient p; $L_* = \{z:\ f_*(x, y) = 0\}$, $L^* = \{z:\ f^*(x, y) = 0\}$ is the external boundary and correspondingly wellbore boundary; $\rho = \rho(p, T)$, $\mu = \mu(p, T)$ is the gas density and viscosity; T is the temperature; k is the absolute medium permeability; χ is the coefficient, characterizing dependence of sedimentary rock permeability (in complicated geological conditions of filtration for which k/μ is a small value) upon pressure gradient, which is determined by the following correlation

$$\chi(I, I_{cr}) = \begin{cases} 1 + F(I - I_{cr}), & \text{if } \ I > I_{cr}, \\ 1, & \text{if } \ I \le I_{cr}, \end{cases} \qquad (2)$$

where F is the specified steadily increasing function; I_{cr} is the critical gradient value.

It must be mentioned that presented as (1) with the consideration of (2) motion equation describes non-linear filtration of fluid or gas in porous medium. That presents the possibility to use a complex variable function for potential flows modelling [1, 6].

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Paying no regard to heat conduction and adiabatic effect, temperature field is found as a solution to the next differential equation with correspondent initial and boundary conditions [7]:

$$\frac{\partial T(x,y,t)}{\partial t}+\boldsymbol{u}(x,y)\big(\boldsymbol{grad}\ T(x,y,t)+ \\ +\varepsilon\ \boldsymbol{grad}\ \ p(x,y)\big)=0, \tag{3}$$

$$T(x,y,0)=T_0(x,y)\ ,\ T(x,y,t)\big|_{L_*}=T_*, \tag{4}$$

where ε is the Joule–Thomson coefficient; $\boldsymbol{u}(x,y,t)=\dfrac{c\rho}{c_f}\boldsymbol{\upsilon}$ is the speed of heat convection in a formation; c, c_f is the specific heat of gas and correspondingly of the formation, saturated with gas. It must be noted that all the values given in this paper are measured in SI system of coordinated units.

To build the problem solution, speed potential is entered as a Liebenson function [8]

$$\Phi(p)=\Phi_*+k\int_{p}^{p_*}\frac{\rho(\tilde p,T)}{\mu(\tilde p,T)}d\tilde p$$

and in accordance with it the system of equation is rewritten (1) with correspondent boundary conditions:

$$div\big(\chi(\tilde I,I_{cr})\,\boldsymbol{grad}\ \Phi\big)=0\ ,$$

$$\boldsymbol{\upsilon}=\frac{\chi(\tilde I,I_{cr})}{\tilde\rho(\Phi)}\boldsymbol{grad}\ \Phi\ , \tag{5}$$

$$\Phi\big|_{L_*}=\Phi(P_*)=\Phi_*\ ,\ \ \Phi\big|_{L^*}=\Phi(P^*)=\Phi^*\ \ (\Phi_*<\Phi^*),$$

where $\tilde\rho(\Phi)=\rho(p(\Phi))$, $\tilde I=\dfrac{\mu}{k\tilde\rho(\Phi)}\sqrt{\Phi_x^2+\Phi_y^2}$.

In this case heat conduction problem (3)–(4) is as follows:

$$\frac{\partial T(x,y,t)}{\partial t}+\frac{c\chi(\tilde I,I_{cr})}{c_{pl}}\boldsymbol{grad}\ \Phi(x,y)\Big(\boldsymbol{grad}\ T(x,y,t)- \\ -\frac{\varepsilon\mu}{k\tilde\rho(\Phi)}\boldsymbol{grad}\ \Phi(x,y)\Big)=0, \tag{6}$$

$$T(x,y,0)=T_0(x,y)\ ,\ T(x,y,t)\big|_{L_*}=T_*\ . \tag{7}$$

To simplify provisions of the main procedure of investigation of thermodynamic effects in a formation wellbore zone that appear as a result of gas throttling through shale rock micro fractures with consideration of geologically complicated conditions, the radial flow will be reviewed (Fig. 1) if $\mu=idem$.

This procedure will be built by disturbance of initial distribution of values $\Phi=\Phi^{(0)}(r)$, $\tilde I=\tilde I^{(0)}(r)$, $Q=Q^{(0)}$ and $r_c=r_c^{(0)}$ (Q is the well rate), which characterize gas extraction process and are obtained from the solution of the correspondent problem in undeformed formation:

$$\frac{d}{dr}\left(\chi^{(0)}r\frac{d\Phi}{dr}\right)=0\ ,$$

$$\Phi\big|_{r=r_0}=\Phi^*\ ,\ \Phi\big|_{r=R_0}=\Phi_*\ , \tag{8}$$

$$\Phi^{(0)}(r)=\Phi_*+\eta\ln\frac{r}{R_0}\ ,$$

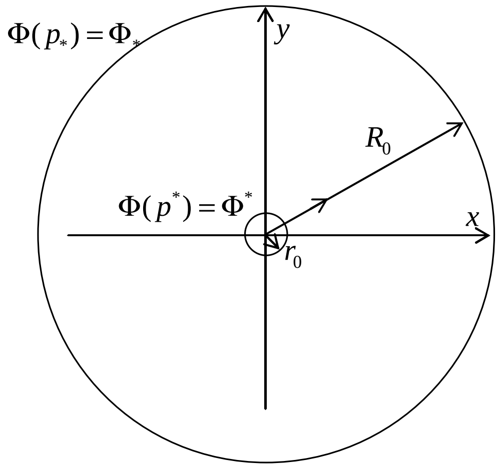

Figure 1 – Radial formation scheme

$$\tilde I^{(0)}(r)=\frac{\mu}{k\tilde\rho(\Phi^{(0)})}\frac{d\Phi^{(0)}}{dr}=\frac{\mu}{k\tilde\rho(\Phi^{(0)})}\frac{\eta}{r}\ ,$$

$$Q^{(0)}=\frac{2\pi\chi_0\eta}{\tilde\rho(\Phi^*)}\ , \tag{9}$$

where $\eta=\dfrac{\Phi^*-\Phi_*}{\ln r_0/R_0}$; $\Phi^*-\Phi_*=\dfrac{k}{\mu}\displaystyle\int_{p^*}^{p_*}\rho(\tilde p)d\tilde p$;

$\chi=\chi^{(0)}=1$ is the coefficient of undeformed medium permeability. Initial approximation of value of r_c , dividing formation into disturbed $[r_0,r_c]$ and undisturbed $[r_c,R_0]$ zones, can be obtained as a solution to equation $\tilde I^{(0)}(r_c^{(0)})=I_{cr}$, which in detail is as follows:

$$r_c^{(0)}\tilde\rho(\Phi^{(0)}(r_c^{(0)}))=\frac{\mu\eta}{kI_{cr}}$$

and depends on the choice of analytical dependence for function $\tilde\rho=\tilde\rho(\Phi(r))$.

Taking into consideration the solution (9) of the problem (8), (5) can be rewritten as:

$$\frac{d}{dr}\left(\chi(I,I_{cr})r\frac{d\Phi(r)}{dr}\right)=0\ ,\ r_0\le r\le R_0\ ,$$

$$\Phi(r_0)=\Phi^*,\ \Phi(R_0)=\Phi_*\ . \tag{10}$$

On the basis of idea of gradual fixation of process and medium characteristics [1–4] solution (10) will be built by iterative correction of problem solution (8).

Then if $F(I-I_{cr})=\alpha(I-I_{cr})=\alpha(\tilde I-I_{cr})$ we will get:

$$\frac{d}{dr}\left(\left(1+\alpha\left(\tilde I^{(0)}-I_{cr}\right)\right)r\frac{d\Phi_1^{(1)}(r)}{dr}\right)=0\ ,\ r_0\le r\le r_c^{(0)}\ ,$$

$$\frac{d}{dr}\left(r\frac{d\Phi_2^{(1)}(r)}{dr}\right)=0\ ,\ r_c^{(0)}\le r\le R_0\ ,$$

$$\Phi_1^{(1)}(r_0)=\Phi^*,\ \Phi_2^{(1)}(R_0)=\Phi_*\ ,$$

$$[\Phi(r)]\big|_{r=r_c^{(0)}}=0\ ,\ [\upsilon_n]\big|_{r=r_c^{(0)}}=0\ .$$

The first approximation of the solution will be as follows:

$$\Phi^{(1)}(r)=\begin{cases}\Phi_1^{(1)}(r)=\Phi^*-\tilde{\Phi}(r_c^{(0)})\ln\dfrac{\alpha I_{cr}\left(r_c^{(0)}-r_0\right)+r_0}{\alpha I_{cr}(r_c^{(0)}-r)+r}, \\ \qquad r_0\le r\le r_c^{(0)}, \\ \Phi_2^{(1)}(r)=\Phi_*+(1-\alpha I_{cr})\tilde{\Phi}(r_c^{(0)})\ln\dfrac{r}{R_0}, \\ \qquad r_c^{(0)}\le r\le R_0;\end{cases}$$

$$\tilde{I}^{(1)}=\frac{\mu}{k\tilde{\rho}(\Phi^{(1)})}\frac{d\Phi^{(1)}}{dr}=$$

$$=\begin{cases}\tilde{I}_1^{(1)}=\dfrac{\mu}{k\tilde{\rho}(\Phi_1^{(1)})}\dfrac{(1-\alpha I_{cr})\tilde{\Phi}(r_c^{(0)})}{\alpha I_{cr}(r_c^{(0)}-r)+r}, r_0\le r\le r_c^{(0)}, \\ \tilde{I}_2^{(1)}=\dfrac{\mu}{k\tilde{\rho}(\Phi_2^{(1)})}\dfrac{(1-\alpha I_{cr})\tilde{\Phi}(r_c^{(0)})}{r}, r_c^{(0)}\le r\le R_0;\end{cases}$$

$$Q^{(1)}=\frac{2\pi}{\tilde{\rho}(\Phi_2^{(1)})}(1-\alpha I_{cr})\tilde{\Phi}(r_c^{(0)}),$$

$$r_c^{(1)}\tilde{\rho}(\Phi_2^{(1)}(r_c^{(1)}))=\frac{\mu(1-\alpha I_{cr})\tilde{\Phi}(r_c^{(0)})}{kI_{cr}},$$

where $\tilde{\Phi}(r)=\dfrac{\Phi^*-\Phi_*}{(1-\alpha I_{cr})\ln\dfrac{r}{R_0}+\ln\left(\alpha I_{cr}+(1-\alpha I_{cr})\dfrac{r_0}{r}\right)}$.

The same entry format we have in the general case to get $n-$approximation:

$$\frac{d}{dr}\left(\left(1+\alpha\left(\tilde{I}_2^{(n-1)}-I_{cr}\right)\right)r\frac{d\Phi_1^{(n)}(r)}{dr}\right)=0,\ r_0\le r\le r_c^{(n-1)};$$

$$\frac{d}{dr}\left(r\frac{d\Phi_2^{(n)}(r)}{dr}\right)=0,\ r_c^{(n-1)}\le r\le R_0;$$

$$\Phi_1^{(n)}(r_0)=\Phi^*,\ \Phi_2^{(n)}(R_0)=\Phi_*,$$

$$[\Phi(r)]\big|_{r=r_c^{(n-1)}}=0,\ [\upsilon_n]\big|_{r=r_c^{(n-1)}}=0;$$

$$\Phi^{(n)}(r)=\begin{cases}\Phi_1^{(n)}(r)=\Phi^*-\tilde{\Phi}(r_c^{(n-1)})\ln\dfrac{\alpha I_{cr}\left(r_c^{(n-1)}-r_0\right)+r_0}{\alpha I_{cr}(r_c^{(n-1)}-r)+r}, \\ \qquad r_0\le r\le r_c^{(n-1)}, \\ \Phi_2^{(n)}(r)=\Phi_*+(1-\alpha I_{cr})\tilde{\Phi}(r_c^{(n-1)})\ln\dfrac{r}{R_0}, \\ \qquad r_c^{(n-1)}\le r\le R_0;\end{cases}$$

$$\tilde{I}^{(n)}=\frac{\mu}{k\tilde{\rho}(\Phi^{(n)})}\frac{d\Phi^{(n)}}{dr}=$$

$$=\begin{cases}\tilde{I}_1^{(n)}=\dfrac{\mu}{k\tilde{\rho}(\Phi_1^{(n)})}\dfrac{(1-\alpha I_{cr})\tilde{\Phi}(r_c^{(n-1)})}{\alpha I_{cr}(r_c^{(n-1)}-r)+r}, r_0\le r\le r_c^{(n-1)}, \\ \tilde{I}_2^{(n)}=\dfrac{\mu}{k\tilde{\rho}(\Phi_2^{(n)})}\dfrac{(1-\alpha I_{cr})\tilde{\Phi}(r_c^{(n-1)})}{r}, r_c^{(n-1)}\le r\le R_0;\end{cases}$$

$$Q^{(n)}=\frac{2\pi}{\tilde{\rho}(\Phi_2^{(n)})}(1-\alpha I_{cr})\tilde{\Phi}(r_c^{(n-1)}),$$

$$r_c^{(n)}\tilde{\rho}(\Phi_2^{(n)}(r_c^{(n)}))=\frac{\mu(1-\alpha I_{cr})\tilde{\Phi}(r_c^{(n-1)})}{kI_{cr}},$$

where $n=2,3,\dots$.

Using already obtained formulas, the problerm of searching of temperature field will be as follows:

$$\frac{\partial T(r,t)}{\partial t}+\frac{c\chi(\tilde{I},I_{cr})}{c_f}\frac{d\Phi^{(n)}(r)}{dr}\left(\frac{dT(r,t)}{dr}-\right.$$
$$\left.-\frac{\varepsilon\mu}{k\tilde{\rho}(\Phi)}\frac{d\Phi^{(n)}(r)}{dr}\right)=0, \tag{11}$$

$$T(r,0)=T_0(r),\ T(r,t)\big|_{L_*}=T_*. \tag{12}$$

We will look for the equation solution (11) by method of characteristics:

$$\left.\frac{dT}{dt}\right|_{r=\xi(t)}=-\frac{c\chi(\tilde{I},I_{cr})}{c_f}\frac{d\Phi^{(n)}(r)}{dr}\left(\frac{\partial T}{\partial r}-\right.$$
$$\left.-\frac{\varepsilon\mu}{k\tilde{\rho}(\Phi)}\frac{d\Phi^{(n)}(r)}{dr}\right)+\left.\frac{\partial T}{\partial r}\right|_{r=\xi(t)}\frac{d\xi}{dt};$$

$$\left.\frac{dT}{dt}\right|_{r=\xi(t)}=\frac{\partial T}{\partial r}\left(\frac{d\xi}{dt}-\frac{c\chi(\tilde{I},I_{cr})}{c_f}\frac{d\Phi^{(n)}(r)}{dr}\right)+$$
$$+\frac{c\varepsilon\mu\chi(\tilde{I},I_{cr})}{c_f k\tilde{\rho}(\Phi)}\left(\frac{d\Phi^{(n)}(r)}{dr}\right)^2,$$

where $\dfrac{d\xi}{dt}=\dfrac{c\chi(\tilde{I},I_{cr})}{c_f}\dfrac{d\Phi^{(n)}(r)}{dr}$ is the differential equation for characteristic; $\left.\dfrac{dT}{dt}\right|_{r=\xi(t)}=\dfrac{c\varepsilon\mu\chi(\tilde{I},I_{cr})}{c_f k\tilde{\rho}(\Phi)}\left(\dfrac{d\Phi^{(n)}(r)}{dr}\right)^2$ is the differential equation to determine the temperature along characteristic.

By substituting values in the above obtained equations $\dfrac{d\Phi^{(n)}(r)}{dr}$ for each of the zones $\left[r_0,r_c^{(n)}\right]$, $\left[r_c^{(n)},R_0\right]$ we will get systems of differential equations, which are solved analytically, depending on certain function type $\tilde{\rho}=\tilde{\rho}(\Phi(r))$, by the known methods [3, 9]

$$\begin{cases}\dfrac{d\xi}{dt}=\dfrac{c\chi(\tilde{I}_1^{(n)},I_{cr})}{c_f}\dfrac{(1-\alpha I_{cr})\tilde{\Phi}(r_c^{(n-1)})}{\alpha I_{cr}(r_c^{(n-1)}-r)+r},\ r_0\le r\le r_c^{(n-1)}, \\ \left.\dfrac{dT}{dt}\right|_{r=\xi(t)}=\dfrac{c\varepsilon\mu\chi(\tilde{I}_1^{(n)},I_{cr})}{c_f k\tilde{\rho}(\Phi_1^{(n)})}\left(\dfrac{(1-\alpha I_{cr})\tilde{\Phi}(r_c^{(n-1)})}{\alpha I_{cr}(r_c^{(n-1)}-r)+r}\right)^2;\end{cases} \tag{13}$$

$$\begin{cases}\dfrac{d\xi}{dt}=\dfrac{c}{c_f}\dfrac{(1-\alpha I_{cr})\tilde{\Phi}(r_c^{(n-1)})}{r},\quad r_c^{(n-1)}\le r\le R_0, \\ \left.\dfrac{dT}{dt}\right|_{r=\xi(t)}=\dfrac{c\varepsilon\mu}{c_f k\tilde{\rho}(\Phi_2^{(n)})}\left(\dfrac{(1-\alpha I_{cr})\tilde{\Phi}(r_c^{(n-1)})}{r}\right)^2;\end{cases} \tag{14}$$

$$T\big|_{t=0}=0,\ T\big|_{r=R_0}=0,\ [T]\big|_{r=r_c^{(n-1)}}=0,\ [u_n]\big|_{r=r_c^{(n-1)}}=0.$$

Fig. 2 demonstrates the distribution of a potential $\Phi=\Phi(r)$ and gradient value $\tilde{I}=\tilde{I}(r)$ when $\Phi_*=0$, $\Phi^*=1$, $r_0=0.1$, $R_0=100$, $k=0.1\cdot10^{-14}$, $\mu=0.1\cdot10^{-4}$, $I_{cr}=0.0048$, $\alpha=10$ for different cases of n.

In the general case dependencies $\mu=\mu(p,T)$, the algorithm of correspondent problem solution is completed with the procedure of stepwise recording of temperature and pressure fields [1–4, 6].

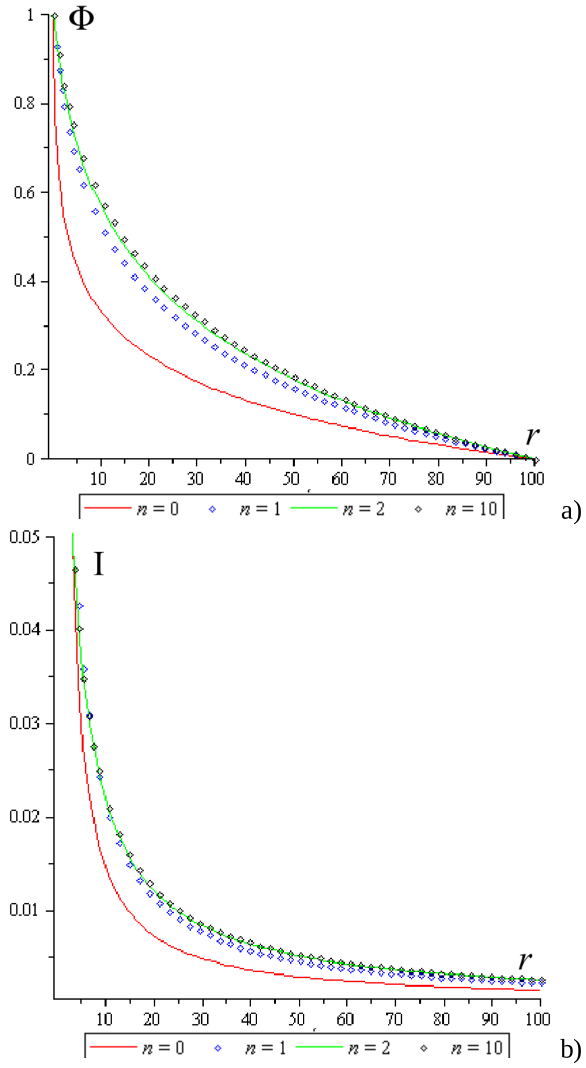

Figure 2 – Distribution of quasipotential values *(a)* and its gradient *(b)* at iterations $n=0$, $n=1$, $n=2$, $n=10$

Conclusion

Thus, modified as (1) and (2), Darcy's law with the critical pressure gradient allows in quasistationary conditions with the use of stepwise recording of process and medium characteristics to model nonlinear filtration and mass-exchanging processes, considering thermodynamic effects which appear as a result of gas throttling through micro fractures of shale rock. That gives the possibility to evaluate effectiveness of use of hydrocarbons production intensification technologies in complicated geological conditions. Correspondent computer experiment taking into account reverse influence of temperature regime upon filtration characteristics is planned to be conducted in future.

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Математичне моделювання термодинамічних ефектів у присвердловинній зоні газового пласта

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З використанням ідей поетапної фіксації характеристик процесу і середовища запропоновано методологію математичного моделювання фільтраційно-масообмінних процесів у присвердловинній зоні пласта з урахуванням термодинамічних ефектів, що виникають внаслідок дроселювання газу через мікротріщини сланцевої породи, коли за умов квазістаціонарності фільтраційної течії процес витіснення описується модифікованим законом Дарсі з критичним градієнтом.

Сформульовано модельну задачу дослідження термодинамічних ефектів у присвердловинній зоні пласта під час фільтрації газу в низькопроникній породі. На прикладі радіальної течії газу проілюстровано особливості побудови алгоритму розподілу температурного поля.

Ключові слова: *ефект Джоуля–Томсона, метод ітерації, нелінійна фільтрація, сланцева осадова порода.*