Mathematical Modeling of Oil Wells Productivity

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The paper describes a mathematical model of determining the potential productivity of oil wells and their current skin effect for most fields in Ukraine. Current skin effect is determined by comparing current productivity with potential productivity of the well. After application of stimulation methods the prospects for production increase and the amount of additional oil production are estimated. The correlation of prospected by the developed method additional oil production and actual production is an evidence of the effectiveness of the proposed model. **Key words:** well, productivity, skin effect, flow capacity, intensification.

In order to make an informed decision on the expediency of applying the methods of intensification of the reservoir fluid inflow to the well, one must first identify the potential performance of layers and the mudding status of the bottomhole zone and only then assess the opportunities and prospects of increasing the production rate by inflow intensification methods.

To solve this problem, the methods of determining the potential productivity and well skin effect, as well as assessing the prospects of the bottomhole zone processing (BZP) were developed [1]. Based on this method, the National Research and Design Institute of Ukrnafta developed *WProduct* software.

The mathematical model of the well potential productivity is based on determination of the water permeability of each productive layer in the well cross-section in particular and all layers in general for layer conditions of deposits of the Precarpathian and Dnieper-Donets depression (DDD). Consequently, the productivity of the well with unmudded bottomhole zone and its flow rate is calculated when the skin effect is zero. Then the current skin effect and the possibility of increasing the flow rate by means of intensification is estimated.

The sequence of determination of the potential well productivity, feasibility and expected efficiency of application of the reinforcement means is as follows:

determine the capacitive power parameter (CPP) of the well, which is the product of the conditional rock capacity and pressure gradient formation;

determine the water permeability factor for each layer, all or selected layers;

calculate the coefficient of potential productivity of each layer, all or selected layers;

calculate the coefficient of actual productivity and the skin effect according to research at steady filtration;

calculate the expected efficiency of the BZP application method in the selected range by skin effect reduction.

Let’s consider the nature of the calculations at each of the aforesaid stages. First, based on the input data on the well the CPP of each productive layer is calculated as a product of the effective layer thickness, porosity determined according to geophysical studies, and layer pressure gradient [2], i.e.:

$$E_j = h_j m_0 \frac{P_{пл}}{H_{пл\, i}}, \qquad (1)$$

where E_j is a capacitive energy parameter m·m³/m³ (MPa/m); h_j is the effective layer thickness, m; m_0 is the mean porosity, m³/m³; P_{PL} is the layer pressure, MPa, H_{PLj} is the average layer perforation depth, m.

CES of all productive layers is determined similarly, taking into account the total effective thickness of all layers and the mean porosity for this thickness.

The threshold CES value is one of the empirical criteria for assessment of the feasibility of applying the methods of intensification of the influx through the BZP, which varies depending on the flow rate required for intensification cost recovery and the expected value of the additional oil recovery. CES threshold is not constant, but depends on the BZP cost prime cost and price of one ton of oil.

The most difficult thing is to determine the reservoir permeability in situ. First, the absolute rock permeability of each reservoir is determined and then recalculated into permeability for oil based on the residual rock water saturation, and in the end It is adjusted by the rock pressure effect on the rock permeability via the compressibility coefficient.

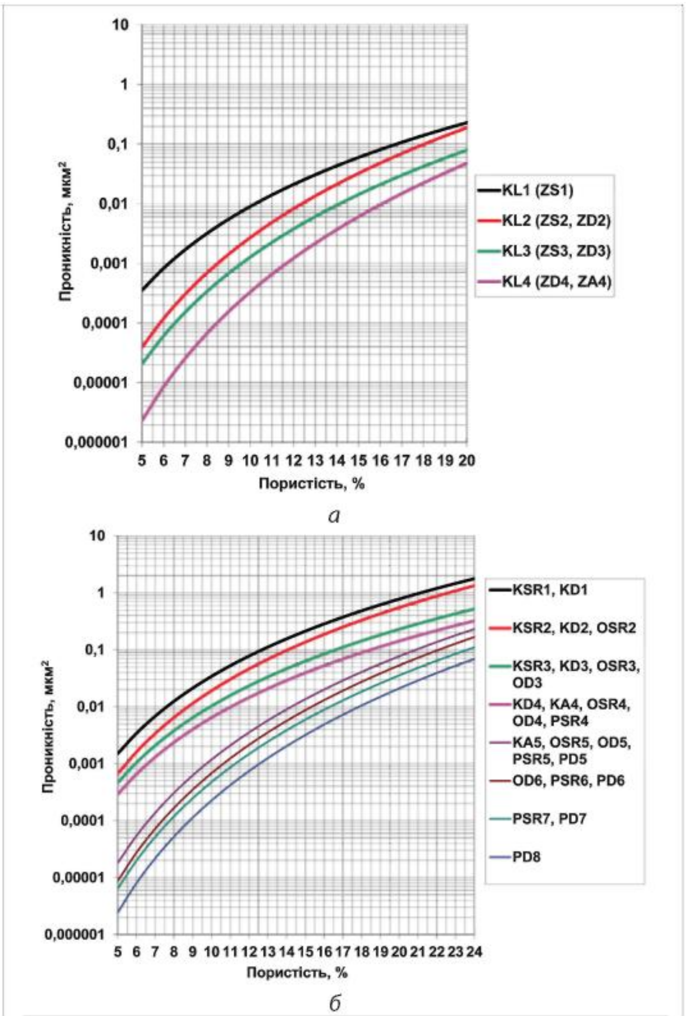


Fig. 1. Dependence of absolute permeability on reservoir porosity and lithotype for Precarpathians *(a)* and DDD *(b)* fields

Table 1

Codes and types of collectors in Precarpathians fields

Particle class and size of clastic grains	Cement contents,%	Collector code types	
		Quartz sandstones	Quartz siltstones
Medium and coarse sandstones (0.25 - 0.5 - 1.0 mm)	Max 5	ZS1	-
	5 - 10	ZS2	-
	10 - 20	ZS3	-
Fine sandstones (0.1 - 0.25 mm)	5	ZD2	-
	5 - 10	ZD3	-
	15 - 20	ZD4	-
Siltstones (0.01 - 0.1 mm)	Max 10	-	ZA

The absolute permeability is determined depending on the lithographic types of collectors allocated based on the fractional composition of grains and cement content. Table 1 and 2 provides information on the types of lithological reservoirs of Precarpathians and DDD by grain size and content of clay and carbonate cement.

The correlation dependencies of absolute permeability from open porosity for each allocated lithotype of Precarpathians and DDD rocks was developed, which became the basis of graphic dependences depicted in Fig. 1.

Then the permeability k_n for reservoir oil filtration at residual water saturation of rock (to approximate the filtration to reservoir conditions for all types of collectors) from absolute permeability k_{and} is calculated based on experimental data, based on which the graphic dependences $_n\ k = f\,(k_{\ a)}$ shown in Fig. 2 were built.

The impact of vertical rock pressure increase on the permeability of rocks for oil in situ is calculated through the compressibility factor $k_{np:}$

$$k_{нр} = k_н \cdot \alpha_{гп}, \qquad (2)$$

where k_n is permeability, $m^2 \bullet 10^{-3;}$ α_{GA} is the rock compressibility factor.

To assess the influence of the rock compression stress effect on permeability, first it is required to determine the rock compression stress as the difference of the vertical mining and reservoir pressure, MPa:

$$\Delta P_{пл} = 0{,}025 H_{пл} - P_{пл}, \qquad (3)$$

where $H_{пл}$ is the average depth of layers, m

The adjustment to reflect the impact of the mining and reservoir pressure on the compressibility of rocks is made based on empirical dependencies shown in Fig. 3, which are constructed using the results of research performed by F. Kotyahov and T. Dahkylgov [3]. The impact of porosity, cement content and tension in the rock, which depends on the difference between vertical mining and reservoir pressure, is taken into account.

Using the graphs shown in Fig. 3, the rock compressibility factor is determined depending on the well depth at the stress corresponding to the reservoir pressure equal to 50, 75 and 100% of the hydrostatic pressure and via the collector type based on the grain size, porosity and its clayness.

The coefficient of permeability of all oil saturated or selected layers shall be determined as the mean by layer thickness [4]:

$$k_{jнр_{сер}} = \frac{\Sigma(k_{jнр} \cdot h_j)}{\Sigma h_j}, \qquad (4)$$

where $k_{jнр}$ is the coefficient of permeability of the *j-th* layer, $m^{2.}$

The water conductivity of each oil saturated layer ε_j taking into account the permeability, effective thickness of reservoirs and oil viscosity at reservoir conditions shall be determined by the well-known formula:

$$\varepsilon_j = \frac{k_{jнр} \cdot h_j}{\mu_j}, \qquad (5)$$

where μ_j is the oil viscosity in reservoir conditions, mPa·s.

The water conductivity of all oil saturated or selected layers is determined as the sum of water permeability.

The well flow rate and productivity shall be determined based on the classical model of radial planar flow to the well of a single-phase Newtonian fluid (oil) according to the Darcy law. On the basis of this law, the flow rate can be calculated as follows:

$$Q = \frac{5{,}43\, kh (P_{пл} - P_{вб})}{\mu b \ln(R_к / r_c) + S}, \qquad ($$

where Q is the liquid flow rate, m^3/day; k is the averaged permeability coefficient, m^2; h is the thickness of layer, m; $P_{вб}$ is the acking pressure, MPa; μ is the oil viscosity in situ, mPa.s; b is the volume ratio of oil; R_k is the feeding circuit radius, m; r_c is the well radius, m; S is the skin effect, which takes into account all the additional resistance (pressure loss) in the well bottomhole zone in a generalized manner.

To calculate the potential values of well productivity and flow rate, i.e. the hydrodynamically perfect well, the skin effect equal to zero *(S*=0) is taken, and the water conductivity value is determined above, so formula (6) will acquire the following form:

$$Q = \frac{5{,}43\varepsilon_{п}(P_{пл} - P_{вб})}{b\ln(R_{к}/r_{с})}. \qquad (7)$$

Table 1 2

Codes and types of collectors at DDD deposits

Particle class and size of clastic grains	Cement content,%	Collector type codes			
		Quartz sandstones	Oligomictic sandstones	Polimictic sandstones	Quartz siltstones
Medium and coarse sandstones	5	-	OSR2	PS R4	-
	5 - 10	KSR1	OSR3	PSR5	-
	10 - 15	KSR2	OSR4	PSR6	-
(0.25 - 0.5 - 1.0 mm)	15 - 20	KSR3	OSR5	PSR7	-
Fine-grained sandstones (0.1 - 0.25 mm)	5	KD1	OD3	PD5	-
	5 - 10	KD2	OD4	PD6	-
	10 - 15	KD3	OD5	PD7	-
	15 - 20	KD4	OD6	PD8	-
Large and various-grained siltstones	5	-	-	-	KA4
	5 - 10	-	-	-	KA5
(0.01 - 0.1 mm)	10 - 15	-	-	-	-

If the reservoir developed the dissolved gas mode, i.e. the reservoir pressure is lower than the saturation pressure $P_{нас}$, it is stipulated to take into account the phase permeability reduction for oil through a part of the depression moving the aerated oil in the reservoir. To take into account the impact of oil degassing subject to reservoir pressure reduction on its influx to the well the industrial analog of Khrystianovych functions *(H),* suggested by I.D. Amelin, was used, which is best determined from the industrial data by the formula Δ H = A·ΔP, where ΔH is the depression on the layer expressed in terms of the Khrystianovych function, MPa; A is a coefficient which depends on the pressure drop degree in the reservoir vs. the saturation pressure; ΔP is a complete depression on the layer, MPa.

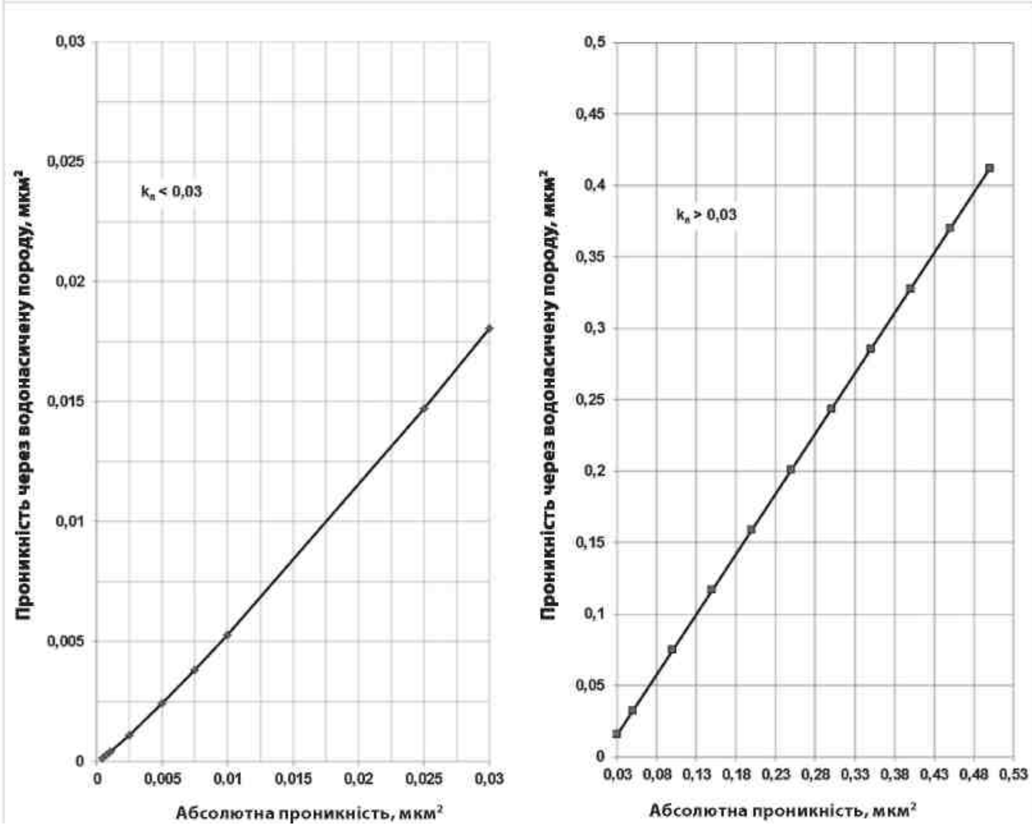


Fig. 2. Dependence of permeability through the water saturated rock from absolute permeability

In terms of Precarpathians deposits the phase permeability reduction was established through a part of the depression which drives the aerated oil in the reservoir by R.V. Mysiovych and V.P. Klyarovskyi who built the graphical dependence $A=f(P_{пл}/P_{нас})$ to determine A coefficient. To assess the impact of reduced reservoir pressure versus saturation pressure $P_{нас}$ on the oil influx to the well we [5] found the correlation as follows:

$$A = 0{,}097e^{(2{,}37 P_{пл}/P_{нас})}. \qquad (8)$$

Table 3

Comparison of predicted productivity and flow rate indicators after BFZ vs. the actual indicators

Well	Productivity coefficient m³/(d · MPa)	Liquid flow rate, m³/day	Productivity coefficient m³/(d · MPa)	Liquid flow rate, m³/day	Productivity coefficient m³/(d · MPa)	Liquid flow rate, m³/day	Productivity coefficient m³/(d · MPa)	Liquid flow rate, m³/day
	before BZP		potential		predicted		after BZP	
1600 – Boryslav	0.57	2.7	1.9	9.5	1.15	5	0.8	4.5
1700 - Boryslav	0.8	3.2	3.9	15	1.8	8	1.14	5.3
73 – Staryi Sambir	0.69	5.7	1.5	14.5	1.35	12, 5	1.34	12, 5
12 - Mr - Bytkiv	0.36	1.7	0.9	4.9	0.8	4.5	0.45	2.5
188 – North Dolyna	0.4	2.6	3.2	23	2.9	15	3.38	23.6
214 - Kachanivka	0.68	2.4	2.2	8.9	1.6	6	1.9	8.2
81 - Reshetnyany	0.51	0.2	2.9	20	1.67	8	2.5	12, 5
On the average per well	0.57	2.64	2.5	13.7	1.46	8.5	1.64	9.7

To take account the gas-saturated oil movement, i.e. when the saturation pressure is greater than the reservoir pressure, formula (7) shall be supplemented with A factor. The remaining parameters should be added to Dupoi formula is the form corresponding to the saturation pressure, and its expression will appear as follows:

$$Q_{п.газ.н} = \frac{5{,}43\varepsilon_п (P_{пл} - P_{вб})}{b \ln(R_к / r_с)} A. \qquad (9)$$

Next, the potential productivity of wells shall be determined:

$$\eta_{п} = \frac{Q_{п}}{P_{пл} - P_{вб}}. \qquad (10)$$

The potential productivity ratio of all layers is the sum of the determined productivity coefficients of individual layers, similarly to the water conductivity. The potential production rate shall be calculated according to the formula (7) or (9).

The actual productivity for the known current values of the reservoir and bottomhole pressures and the fluid flow rate shall be calculated using the following formula (11). For comparison, you can set some of such values obtained from the studies and measurements at the well during its operation.

$$\eta_{ф} = \frac{Q_{ф}}{P_{пл} - P_{вб}}. \qquad (11)$$

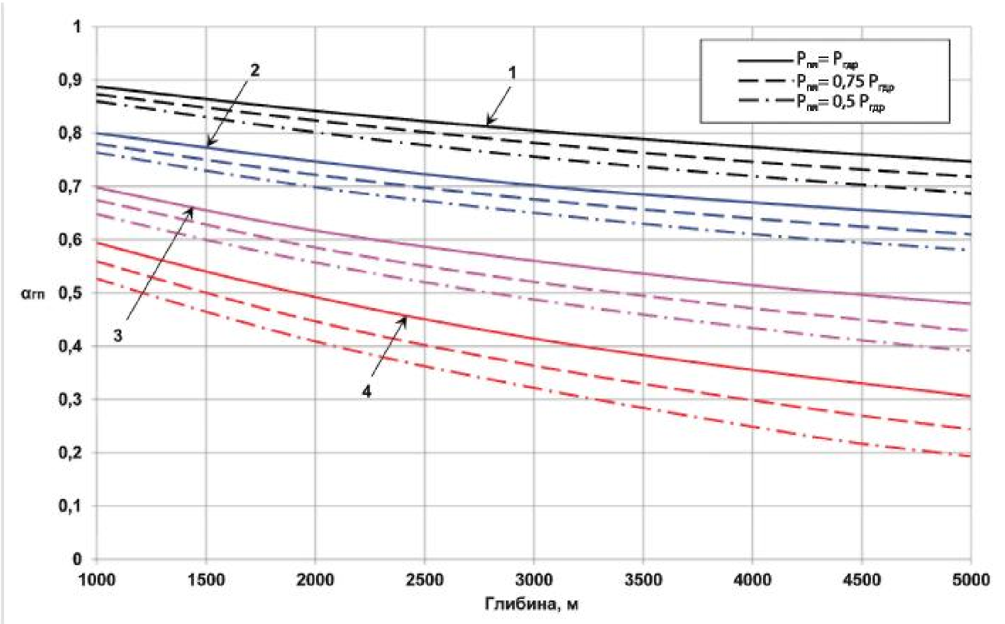
To determine the current skin effect of the wells, the ratio of *ВП* productivities shall be established first:

$$ВП = \frac{\eta_{ф}}{\eta_{п}}. \qquad (12)$$

The magnitude of the *S* skin effect for each measurement of the well productivity based on $R_к$ and $r_с$ values and the found *ВП* value shall be calculated by the formula:

$$S = \frac{\ln\left(R_{К}/r_{С}\right)}{ВП} - \ln\left(R_{К}/r_{С}\right). \qquad (13)$$

The calculations of BZP efficiency shall be performed depending on changes in the skin effect and the magnitude of the depression on the reservoir. The expected performance and flow rate after BZP sha;; be calculated by the formula (6) for the skin effect set by the user taking into account the experience of using a particular BZP method. For example, after the acid fracturing (KGRP) the achieved skin effect *S*=2...-1 is lower than after the application of strong fracturing (PHRP), which is the strongest of the known methods of intensification and depending on the size and conductivity of the crack it provides a reduced skin effect to *S*=-1...-3.



1 - The first group of collector codes: ZS1, ZD2 (Precarpathians), KSR1, OSR2, OSR3 (PPD);

2 - The second group of collector codes: ZS 2, ZD 3 (Precarpathians), KSR 2, KSR 3, KD 1, OSR 4, OSR 5, OD 3 (DDD);

3 - The third group of collector codes: ZS 3, Z A (Precarpathians), KD 2, KD 3, KD 4, OD 4, OD 5, PSR 4, PSR 5, PD 5 (PPD);

4 - The fourth group of collector codes: ZD 4 (Precarpathians), OD 6, PSR 6, PSR 7, PD 6, PD 7, PD 8, KA 4, KA 5 (PPD).

Fig. 3. Dependence of the rock compressibility ratio on its occurrence depth

The described mathematical model for determination of wells productivity is the basis of *WProduct* [6] application, which is used for oil wells of the deposits of UkrNafta PJSC on the stage of selecting objects for the purpose of carrying out PGRP and KGRP.

Table 3 provides the projected flow rate and productivity values prior to the application of intensification measures determined using the software and the actual data on wells operation after PGRP and KGRP, for comparison.

From the data in the above Table 3 it can be seen that:

The current productivity rate of all wells was significantly (almost four times) lower than the potential, i.e. there were good prerequisites for the use of hydrocarbon influx intensification methods;

The projected rates of productivity and flow rate after the planned BZP is lower than the potential, since the intensification methods were planned only for a part of the reservoir wells;

The average productivity factor of seven wells after BZP differs from the predicted factor by less than 15%.

The developed techniques is used for annual modeling of the potential productivity and the expected additional oil production in about 100 wells of UkrNafta PJSC fields on the stage of selecting sites for EMG. The average additional production coincides with the expected one, indicating the effectiveness of the proposed technique.

Conclusion

Therefore, the mathematical model of determining the potential productivity of oil wells, which are used effectively to support the expediency of well BZP in the fields of UkrNafta PJSC was developed.

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