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Forced oscillations of an oil pipeline at an overhead crossing during sequential pumping of various oil products

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Abstract. The method of sequential pumping of various oil products separated by a diaphragm seal is one of the most common and cost-effective. However, due to the difference in densities, significant oscillations of oil pipelines occur in their overhead part. There are practically no studies of such oscillations and their impact on the strength and stability of overhead oil pipelines, which makes this topic relevant. The aim of this article was to determine the oscillations of the oil pipeline axis and the bending moments that occur at this time, without taking into account the inertial forces of the pumped oil products for a single-span beam crossing without longitudinal deformation compensators. A mathematical model of the sequential pumping of two different oil products through a pipeline was developed. The problem was solved by decomposing the desired solution into a series of eigenfunctions of the problem of free oscillations of the overhead part of the oil pipeline using the Fourier method. As a result of calculations based on the obtained solution to the problem, it was found that in the overhead part of the oil pipeline, during the sequential pumping of various oil products, there are familiar oscillations of the oil pipeline axis relative to the abscissa axis of the deflection function of the oil pipeline axis. When the end cross-sections of the diaphragm seal enter the overhead part of the pipeline, the largest deflections occur in one of the middle sections of the part in tenths of a second. The bending moments have the highest values by modulus. The deflection curves of the oil pipeline axis and the bending moment curves for these moments of time practically coincide. It was found that the largest modulus bending moments of the pipeline during the sequential pumping of two different oil products are significantly higher than the bending moments of the same oil

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products when they are pumped separately. The obtained results of the study will be useful in practice for designers of pipelines that will be used to pump oil products with different densities

Keywords: single-span beam crossing; pipe axis; eigenfunctions of the free oscillations problem; deflections of the pipeline axis; separating medium; bending moment

Introduction

Overhead parts of oil pipelines are among the most heavily loaded parts. In addition to the internal pressure of the pumped product, they are exposed to changes in the ambient temperature regime (daily, seasonal). The weight of the pipeline itself and the pumped oil products and the forces arising from their movement cause deflections of the pipeline axis. The topography of the area where the pipeline is laid (plain, mountainous), the length of the overhead part, and soil types require consideration of the interaction of the pipeline with the soil and its supports. In 2018-2023, a number of papers appeared that investigated some of the above problems, for example, M. Dutkiewicz *et al.* (2023) investigated the interaction of an oil pipeline with its support built in a mountainous area. With the aim of taking into account the properties of the soil base of the crossing on the strength of the transition, a mechanical and mathematical model of an overhead crossing of an oil pipeline laid in a mountainous area was developed in the study by A. Velychkovych *et al.* (2019). On its basis, simple analytical results were obtained that are suitable for practical engineering calculations. The analysis of the stress state in a damaged composite-coated pipeline was the subject of a study by T. Fan *et al.* (2022).

As noted by Y. Ding *et al.* (2021) and J. Yu *et al.* (2022), oil pipelines can be subjected to abnormal loads during construction and operation, and, according to C.N. Vanitha *et al.* (2023), to various risks. Long-term operation of pipelines leads to damage to pipeline steel at the micro level and can contribute to the emergence of macro defects, as described by K.U. Amandi *et al.* (2019). Therefore, the remaining service life of an oil pipeline is often associated with the study of the corrosion situation, as in the paper by M. Bembenek *et al.* (2022). The results of a study of the corrosion situation are used to predict the remaining service life of the pipeline and are a guide for establishing the inspection cycle and maintenance strategy, as indicated by A. Bi *et al.* (2022). Equally important are the issues of safe operation of underground parts of oil pipelines in difficult mining and geological conditions, as addressed by S. Dey & S. Tesfamariam (2022) and M. Dutkiewicz *et al.* (2022). The destruction of a single part of the pipeline can lead to a catastrophic emergency, environmental problems and significant financial losses. Therefore, for pipelines with a long history of operation, the problems of preventing the occurrence of pipeline wall defects are particularly relevant.

As can be seen from the analysis, researchers have not yet determined the forces of inertia of transported products in oil pipelines during their movement and their impact on the strength and stability of open parts of oil pipelines. This also includes the sequential pumping of different oil products through oil product pipelines. This method is cost-effective and quite common in practice. Due to the difference in the densities of the pumped products, this method causes quite significant oscillations in oil pipelines in their open parts, which is why this problem is the subject of this article. The purpose of this work was to study the oscillations of the overhead part of the oil pipeline. To achieve this goal, the following tasks needed to be solved. The first task was to build a mathematical model of the sequential pumping of two different oil products through a pipeline with a diaphragm seal between them. The second task was to find the displacements of the pipeline axis points as different oil products separated by a diaphragm seal pass through the overhead part of the pipeline. The third task was to determine the bending moments caused by the oscillations of the pipeline, which can only be found after determining these oscillations.

Materials and Methods

The deflections of the oil pipeline axis and bending moments during the sequential pumping of various oil products through an overhead pipeline crossing were determined by the analytical method. A single-span beam crossing without longitudinal deformation compensators with clamped ends was considered. The pipeline is made of 13GS steel (TU 14-3-1573-96) with an outer diameter of $D_3 = 529$ mm, wall thickness $\delta = 10$ mm, and tensile and yield strengths of this steel of $\sigma_u = 510$ MPa and $\sigma_y = 360$ MP, respectively. The pumped oil products are petrol and diesel fuel. The density of petrol and diesel fuel is $\rho_p = 750$ kg/m^3 and $\rho_d = 860$ kg/m^3, respectively. The diaphragm seal between petrol and diesel fuel is a rubber piston with a length of $\Delta x = 0.763$ m and a rubber density of $\rho_r = 575$ kg/m^3. The deflections of the pipeline axis were determined without taking into account the inertial forces of the pumped oil products.

Since the problem was solved by the analytical method, it is necessary to note how the mathematical formulation of the problem was formulated. Oil pipeline oscillations arise because oil products and a diaphragm seal with different densities pass through the overhead part of the pipeline. If a single oil product were being pumped, there would be no oscillations in any of the crossings of the pipeline. At the boundaries between the oil products and the diaphragm seal, a density jump occurs, which is the source of oscillations in each part of the pipeline. On the right-hand side of the differential equation are the force loads created by the pumped oil products and the diaphragm seal, which are multiplied by certain unit asymmetric functions (Korn & Korn, 1968), and for the diaphragm seal by the difference of the asymmetric functions.

The asymmetric functions are chosen in such a way that knowing the moment of time from the assumed initial moment and the specific cross-section of the pipeline, it is possible to immediately tell which force factor at a given moment of time creates pressure on a given cross-section of the oil pipeline. The boundary conditions in the problem are determined by the fact that the ends of the overhead part of the pipeline are clamped. This means that the displacements of the endpoints of the axis of the overhead part and the angles of rotation of the sections of the pipeline passing through these points must be zero. At the initial moment of time, the entire part of the pipeline contains only petrol. This determined the initial task of the problem. The problem was solved by expanding the desired solution into a series of eigenfunctions of free oscillations of the part of the oil pipeline (Fridman, 2018). The deflections of the oil pipeline axis in its overhead (open) section (Fig. 1) were determined during the sequential pumping of two different oil products separated from each other by a diaphragm seal.

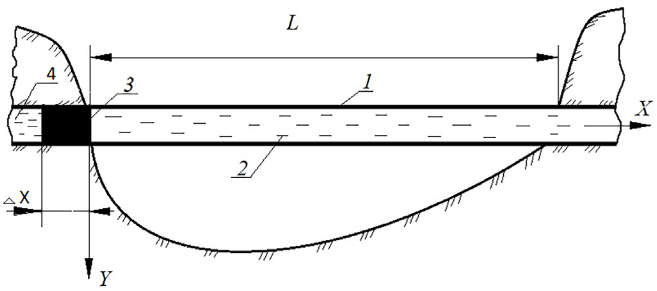


Figure 1. Schematic of an overhead crossing for sequential pumping of two different oil products

Note: 1 – oil pipeline; 2 – pumped petrol; 3 – diaphragm seal; 4 – diesel fuel

Source: created by the authors

Figure 1 shows the moment when the diaphragm seal (its right part) is at the beginning of the abscissa axis X ($x = 0$). The pumped products move at a constant velocity υ. The axial length of the diaphragm seal is Δx. The ends of the open part of the oil pipeline should be assumed to be clamped. In this problem, the displacements of the points of the oil pipeline axis in the direction perpendicular to the initial (horizontal) position of the oil pipeline axis were considered, taking into account the weights of oil products and the diaphragm seal and without taking into account their inertial forces during movement. In this case, the differential equation describing the movement of the points of the pipeline axis has the following form:

$$\frac{\partial^2 u}{\partial t^2} + a^2 \frac{\partial^4 u}{\partial x^4} = \frac{1}{\rho A}\{q_1 S_+(x - vt) + q_2[S_-(vt - x) - S_+(v(t - \Delta t) - x)] + q_3 S_+(v(t - \Delta t) - x)\}. \quad (1)$$

In this case, the initial and boundary conditions will be as follows:

$$u_{/t=0} = \frac{q_1}{2EI}\left(\frac{x^4}{12} - \frac{Lx^3}{6} + \frac{L^2x^2}{12}\right), \frac{\partial u}{\partial t}/_{t=0} = 0; \quad (2)$$

$$u_{/x=0} = 0, \frac{\partial u}{\partial x}/_{x=0} = 0, u_{/x=L} = 0, \frac{\partial u}{\partial x}/_{x=L} = 0, \quad (3)$$

where u is the displacement of the points of the oil pipeline axis in the direction perpendicular to its initial position; $a^2 = EI/(A\rho)$; ρ, A, I, E are the density of the oil pipeline material, its cross-sectional area, axial moment of inertia of the cross-section of the oil pipeline pipes and the elastic modulus of the material; x, t are the coordinate of the oil pipeline axis and time from the beginning of the movement of pumped products in the overhead crossing; q_i, ($i = 1, 2, 3$) are weights per unit length of the distributed load of petrol, diaphragm seal and diesel fuel; υ is the speed of movement of oil products in the overhead crossing; $\Delta t = \Delta x/\upsilon$; L is the length of the overhead crossing; $S_+(x)$ are the asymmetric unit functions (Fridman, 2018).

$$S_-(x) = \begin{cases} 1, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0, \end{cases} \quad S_+(x) = \begin{cases} 1, & \text{if } x > 0, \\ 0, & \text{if } x \leq 0. \end{cases}$$

The created mathematical model was used to find the displacements of the oil pipeline axis points and bending moments of the pipeline overhead crossing part. The Fourier method was used to solve the problem.

Results

The value on the right-hand side of equation (1) in curly brackets is denoted by $q(x, t)$, i.e:

$$q(x, t) = q_1 S_+(x - \upsilon t) + q_2[S_-(\upsilon t - x) - S_+(\upsilon(t - \Delta t) - x)] + q_3 S_+(\upsilon(t - \Delta t) - x). \quad (4)$$

The formulated problem should be solved by decomposing the desired solution into a series of eigenfunctions of the problem of free oscillations of an overhead crossing (Fridman, 2018). To do this, it is necessary to represent $q(x, t)$ as a series:

$$q(x, t) = X_1(x)S_1(t) + X_2(x)S_2(t) + \ldots + X_i(x)S_i(t) + \ldots. \quad (5)$$

In the form of a series, it is necessary to find a solution for deflections (displacements) of the centres of gravity of the pipeline pipe cross-sections (Fourier method):

$$u(x, t) = X_1(x)T_1(t) + X_2(x)T_2(t) + \ldots + X_i(x)T_i(t) + \ldots, \quad (6)$$

where $X_1(x)$, $X_2(x)$, etc. are eigenfunctions in the case of free oscillations of the overhead part of the pipeline; $S_1(t)$,

$S_2(t)$, ..., $T_1(t)$, $T_2(t)$, etc. are unknown functions. Using the Fourier method, it can be shown that the eigenfunctions of this problem are as follows:

$$X_k(x) = K_4(\lambda_k)K_3\left(\frac{\lambda_k}{L}x\right) - K_3(\lambda_k)K_4\left(\frac{\lambda_k}{L}x\right), \qquad (7)$$

where $K_3\left(\frac{\lambda_k}{L}x\right), K_4\left(\frac{\lambda_k}{L}x\right)$ – are Krylov's functions (Filippov, 1965): $K_3\left(\frac{\lambda_k}{L}x\right) = \frac{1}{2}\left(ch\left(\frac{\lambda_k}{L}x\right) - cos\left(\frac{\lambda_k}{L}x\right)\right)$; $K_4\left(\frac{\lambda_k}{L}x\right) = \frac{1}{2}\left(sh\left(\frac{\lambda_k}{L}x\right) - sin\left(\frac{\lambda_k}{L}x\right)\right)$; λ_k – are the roots of the transcendental equation $ch\,cos\,\lambda = 1$. To determine the time function $S_i(t)$, it is necessary to multiply both parts of equation (5) by the eigenfunction (7) and integrate the result over the entire length of the overhead part of the oil pipeline. Due to the orthogonality of the eigenfunctions, only one term remains on the right-hand side, corresponding to the number k, so that:

$$\begin{aligned} S_k(t) = \frac{1}{\int_0^L X_k^2(x)dx}\int_0^L \{q_1S_+(x - vt) + q_2[S_-(vt - x) - \\ -S_+(v(t-\Delta t) - x)] + q_3S_+(v(t-\Delta t) - x)\} \\ \left[K_4(\lambda_k)K_3\left(\frac{\lambda_k}{L}x\right) - K_3(\lambda_k)K_4\left(\frac{\lambda_k}{L}x\right)\right]dx. \end{aligned} \qquad (8)$$

Next, it is necessary to integrate the numerator and denominator of equation (8). The value of the integral in the denominator is known (Filippov, 1965):

$$\int_0^L X_k^2(x)dx = \frac{L}{4}[K_4(\lambda_k)K_1(\lambda_k) - K_3(\lambda_k)K_2(\lambda_k)]^2. \qquad (9)$$

The integrals of the numerator must be found. Once they are found, the following is obtained:

$$\begin{aligned} S_k(t) = \frac{4}{\lambda_k[K_4(\lambda_k)K_1(\lambda_k) - K_3(\lambda_k)K_2(\lambda_k)]^2}\Big\langle q_1\Big\{[K_4^2(\lambda_k) - K_3(\lambda_k)K_1(\lambda_k)] - \left[K_4(\lambda_k)K_4\left(\frac{\lambda_k}{L}vt\right) - K_3(\lambda_k)K_1\left(\frac{\lambda_k}{L}vt\right)\right]\Big\} + \\ +q_2\left\{K_4(\lambda_k)\left[K_4\left(\frac{\lambda_k}{L}vt\right) - K_4\left(\frac{\lambda_k}{L}v(t-\Delta t)\right)\right] - K_3(\lambda_k)\times\left[K_1\left(\frac{\lambda_k}{L}vt\right) - K_1\left(\frac{\lambda_k}{L}v(t-\Delta t)\right)\right]\right\} + \\ +q_3\left[K_4\left(\frac{\lambda_k}{L}v(t-\Delta t)\right) - K_3(\lambda_k)K_3\left(\frac{\lambda_k}{L}v(t-\Delta t)\right)\right]\Big\rangle. \end{aligned} \qquad (10)$$

In order to shorten the writing for further presentation in (10), the following notation should be introduced:

$$\frac{4}{\lambda_k[K_4(\lambda_k)K_1(\lambda_k) - K_3(\lambda_k)K_2(\lambda_k)]^2} = C_1;$$

$$K_4^2(\lambda_k) - K_3(\lambda_k)K_1(\lambda_k) = C_2.$$

Given that each term of series (5) causes a motion described by the corresponding term of series (6), equation (1) can be written in the following form:

$$X_k\ddot{T}_k + a^2X_k^{IV}T_k = \frac{X_kS_k}{\rho A}. \qquad (11)$$

Dividing both parts of equation (11) by $X_k\,T_k$, the following is obtained:

$$-a^2\frac{X_k^{IV}}{X_k} = \frac{\ddot{T}_k}{T_k} - \frac{S_k}{\rho A T_k}. \qquad (12)$$

The left-hand side of equation (12) is equal to $-p_k^2$ (p_k – natural oscillation frequency, $p_k = \frac{\lambda_k^2}{L^2}\sqrt{\frac{EI}{A\rho}}$ (Filippov, 1965). Therefore, the differential equation for the function T_k is as follows:

$$\ddot{T}_k + p_k^2T_k = \frac{S_k}{\rho A}. \qquad (13)$$

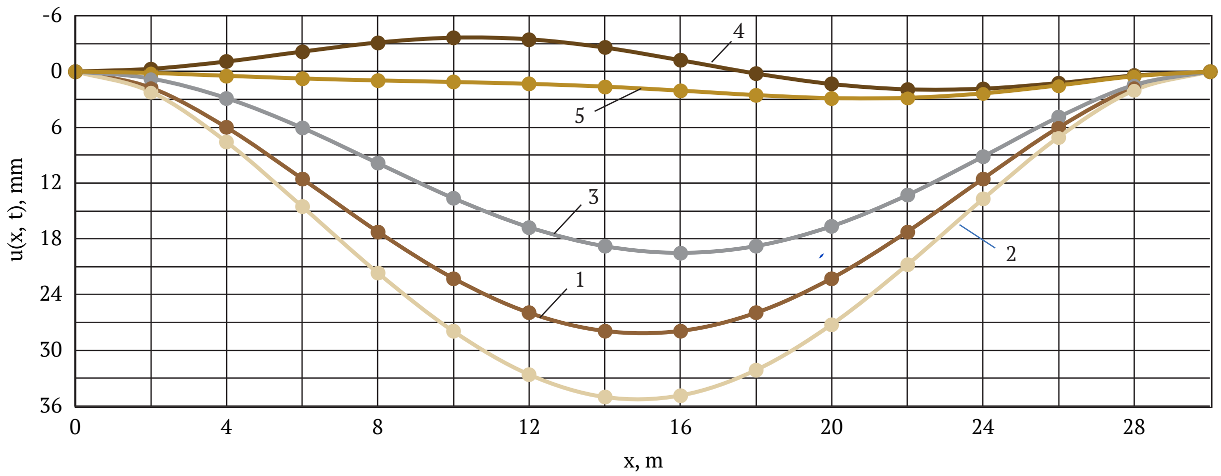
The general solution of equation (13) is represented by the analytical expression:

$$T_k(x) = \frac{1}{\rho A p_k}\int_0^t S_k(\tau)\,sin\,p_k\,(t-\tau)\,d\tau. \qquad (14)$$

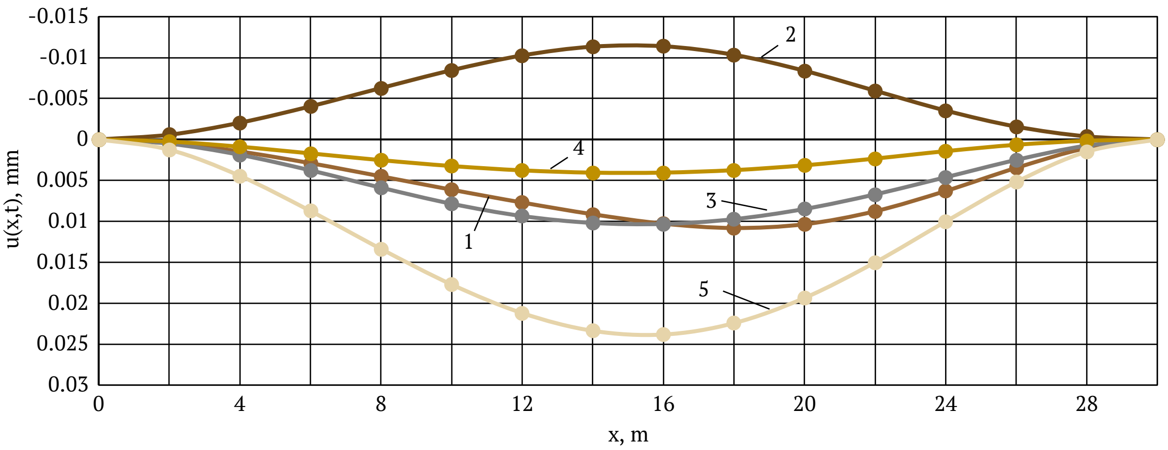
When substituting the analytical expression (10) into (14), it is necessary to perform integration and, using (6), find a solution to problems (1-3). It is necessary to take into account the fact that the number of eigenfunctions is infinitely large. It should also be taken into account that at the initial time ($t = 0$), the pumped petrol fills the entire overhead part of the pipeline and causes a deflection of its axis, which must be determined by revealing the static uncertainty of the overhead part of the pipeline using the method of forces (Gere & Goodno, 2012). In this regard, the solution to problem (1-3) has the following final form:

$$\begin{aligned} u(x,t) = \frac{q_1}{2EI}\left(\frac{x^4}{12} - \frac{Lx^3}{6} + \frac{L^2x^2}{12}\right) + \frac{C_1}{\rho A}\sum_{k=1}^{\infty}\left[K_4(\lambda_k)\times K_3\left(\frac{\lambda_k}{L}x\right) - K_3(\lambda_k)K_4\left(\frac{\lambda_k}{L}x\right)\right]\Bigg\langle q_1\Bigg\{\frac{C_2}{p_k^2}(1 - cos\,p_k\,t) - \\ -K_4(\lambda_k)\left[\frac{sh\left(\frac{\lambda_k}{L}vt\right)}{2\left(\left(\frac{\lambda_k}{L}v\right)^2 + p_k^2\right)} - \frac{\left(\frac{\lambda_k}{L}v\right)^3 sin\,p_kt}{p_k\left(\left(\frac{\lambda_k}{L}v\right)^4 - p_k^4\right)} + \frac{sin\left(\frac{\lambda_k}{L}vt\right)}{2\left(\left(\frac{\lambda_k}{L}v\right)^2 - p_k^2\right)}\right] + K_3(\lambda_k)\left[\frac{ch\left(\frac{\lambda_k}{L}vt\right)}{2\left(\left(\frac{\lambda_k}{L}v\right)^2 + p_k^2\right)} - \frac{cos\left(\frac{\lambda_k}{L}vt\right)}{2\left(\left(\frac{\lambda_k}{L}v\right)^2 - p_k^2\right)} + \frac{p_k^2\,cos\,p_kt}{\left(\left(\frac{\lambda_k}{L}v\right)^4 - p_k^4\right)}\right]\Bigg\} + \\ +q_2\Bigg\{K_4(\lambda_k)\times\left[\frac{sh\left(\frac{\lambda_k}{L}vt\right)}{2\left(\left(\frac{\lambda_k}{L}v\right)^2 + p_k^2\right)} + \frac{sin\left(\frac{\lambda_k}{L}vt\right)}{2\left(\left(\frac{\lambda_k}{L}v\right)^2 - p_k^2\right)} - \frac{\left(\frac{\lambda_k}{L}v\right)^3 sin\,p_kt}{p_k\left(\left(\frac{\lambda_k}{L}v\right)^4 - p_k^4\right)}\right] - K_4(\lambda_k)\left[\frac{sh\left(\frac{\lambda_k}{L}v(t-\Delta t)\right)}{2\left(\left(\frac{\lambda_k}{L}v\right)^2 + p_k^2\right)} + \frac{sin\left(\frac{\lambda_k}{L}v(t-\Delta t)\right)}{2\left(\left(\frac{\lambda_k}{L}v\right)^2 - p_k^2\right)} - \frac{\left(\frac{\lambda_k}{L}v\right)^3 sin\,p_k(t-\Delta t)}{p_k\left(\left(\frac{\lambda_k}{L}v\right)^4 - p_k^4\right)}\right] - \\ -K_3(\lambda_k)\left[\frac{ch\left(\frac{\lambda_k}{L}vt\right)}{2\left(\left(\frac{\lambda_k}{L}v\right)^2 + p_k^2\right)} - \frac{cos\left(\frac{\lambda_k}{L}vt\right)}{2\left(\left(\frac{\lambda_k}{L}v\right)^2 - p_k^2\right)} + \frac{p_k^2\,cos\,p_kt}{\left(\left(\frac{\lambda_k}{L}v\right)^4 - p_k^4\right)}\right] + K_3(\lambda_k)\times\left[\frac{ch\left(\frac{\lambda_k}{L}v(t-\Delta t)\right)}{2\left(\left(\frac{\lambda_k}{L}v\right)^2 + p_k^2\right)} - \frac{cos\left(\frac{\lambda_k}{L}v(t-\Delta t)\right)}{2\left(\left(\frac{\lambda_k}{L}v\right)^2 - p_k^2\right)} + \frac{p_k^2\,cos(p_k(t-\Delta t))}{\left(\frac{\lambda_k}{L}v\right)^4 - p_k^4}\right]\Bigg\} + \\ +q_3\Bigg\{K_4(\lambda_k)\left[\frac{sh\left(\frac{\lambda_k}{L}v(t-\Delta t)\right)}{2\left(\left(\frac{\lambda_k}{L}v\right)^2 + p_k^2\right)} + \frac{sin\left(\frac{\lambda_k}{L}v(t-\Delta t)\right)}{2\left(\left(\frac{\lambda_k}{L}v\right)^2 - p_k^2\right)} - \frac{\left(\frac{\lambda_k}{L}v\right)^3 sin\,p_k(t-\Delta t)}{p_k\left(\left(\frac{\lambda_k}{L}v\right)^4 - p_k^4\right)}\right] - K_3(\lambda_k)\times\left[\frac{ch\left(\frac{\lambda_k}{L}v(t-\Delta t)\right)}{2\left(\left(\frac{\lambda_k}{L}v\right)^2 + p_k^2\right)} + \frac{cos\left(\frac{\lambda_k}{L}v(t-\Delta t)\right)}{2\left(\left(\frac{\lambda_k}{L}v\right)^2 - p_k^2\right)} - \frac{\left(\frac{\lambda_k}{L}v\right)^2 cos\,p_k(t-\Delta t)}{\left(\frac{\lambda_k}{L}v\right)^4 - p_k^4}\right]\Bigg\}\Bigg\rangle. \end{aligned} \qquad (15)$$

Using the formula (15), the deflections $u(x, t)$ of the oil pipeline axis can be determined every 2 m along the overhead part for different time points in the interval from 0 to 15 s, as well as the bending moments $M = -EI\frac{\partial^2 u}{\partial x^2}$. The calculations were performed for the following initial data: $q_1 = 1{,}497$ N/m; $q_2 = 1{,}148$ N/m; $q_3 = 1{,}717$ N/m; $\upsilon = 2$ m/s; $E = 2.05 \times 10^{11}$ Pa; $I = 0.000549$ м^4; $\rho = 7{,}850$ kg/m^3; $\Delta t = 0.3815$ s; $A = 0.016305$ m^2; $L = 30$ m. The results are shown in Figures 2-4.



a)



b)

Figure 2. Displacement of the oil pipeline axis points during sequential pumping of petrol and diesel fuel for different time points

Note: a) 1 = 0 s, 2 = 1 s, 3 = 4 s, 4 = 7 s, 5 = 9 s; b) 1 = 8 s, 2 = 10 s, 3 = 12 s, 4 = 14 s, 5 = 15 s
Source: created by the authors

The movements of the oil pipeline axis point every two metres for different time points from $t = 0$ s to $t = 9$ s are shown in Figure 2a. As can be seen from the graphs, when different oil products are pumped sequentially, there are familiar oscillations of the pipeline axis relative to the *X*-axis caused by the sequential pumping of different oil products. The largest of the displacements shown corresponds to time $t = 1$ s and u_{max}. As time t increases, the deviation of the points from the *X*-axis decreases. Larger deviations from the *X*-axis correspond to positive values of $u(x, t)$, and smaller deviations to negative values. At larger time values (from $t = 8$ s to $t = 15$ s) (Fig. 2b), the oscillation process of the oil pipeline axis continues, but with a smaller deviation from the *X*-axis. Figure 3 shows the bending moment graphs for the same time points as for the displacements in Figure 2. It can be noted that smaller time values correspond to larger modulus values of bending moments. In Figure 3a, for example, curve 2, corresponding to time $t = 1$ s, has the largest modulus bending moment M = −140 kN·m. The largest modulus value of the bending moment at $t > 8$ s is curve 10 (Fig. 3b), which corresponds to time $t = 15$ s and is equal to M = −95.3 kN·m.

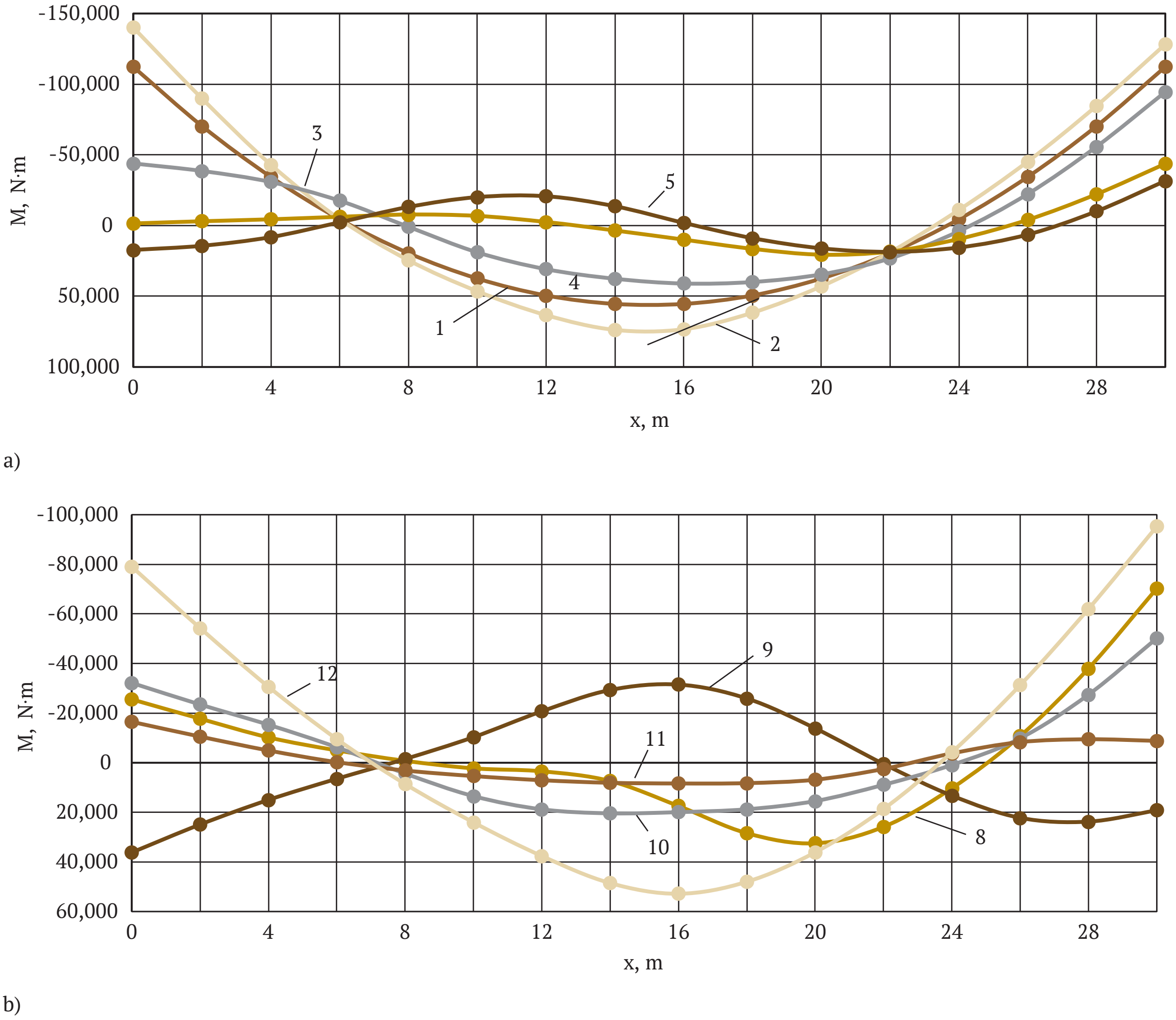


Figure 3. Bending moments of the oil pipeline during sequential pumping of petrol and diesel fuel for different moments of time

Note: a) 1 = 0 s, 2 = 1 s, 3 = 4 s, 4 = 6 s, 5 = 7 s; b) 8 = 8 s, 9 = 10 s, 10 = 12 s, 11 = 14 s, 12 = 15 s
Source: created by the authors

Since small values of time correspond to larger displacements of the points of the oil pipeline axis from the abscissa axis X, the displacements and bending moments at small values of time were calculated. The results are shown in Figure 4. The displacements of the axis of the 30 m long overhead oil pipeline crossing in Figure 4a correspond to the time points $t = 0.3$ s, 0.35 s and 0.45 s. At $t = 0.45$ s, the displacement reaches a maximum equal to u_{max} and the same maximum is obtained at $t = 0.15$ s (this curve is not plotted in Figure 4a, it almost coincides with the displacement curve for $t = 0.45$ s). Bending moments for small values of time are shown in Figure 4b. At $t = 0.15$ s and $t = 0.45$ s, the bending moment curves, similarly to the displacements, almost coincide. The largest modulus value of the bending moments at $t = 0.45$ s is M = −144.7 kN·m, and for $t = 0.45$ s M = −143.6 kN·m, and they correspond to the final cross-section $x = 30$ m of the part of the oil pipeline. It is necessary to compare the largest bending moment modulus for the sequential pumping of petrol and diesel fuel with the largest bending moment modulus of petrol and diesel fuel for their separate pumping. When petrol and diesel are pumped separately, the formula for their bending modulus is:

$$M = EI\frac{\partial^2 u}{\partial x^2} = \frac{q_i}{2}\left(x^2 - Lx + \frac{L^2}{6}\right), i = 1;\ 3;$$
$$q_1 = 1{,}497\,\text{N/m},\ q_3 = 1{,}717\,\text{N/m}. \quad (16)$$

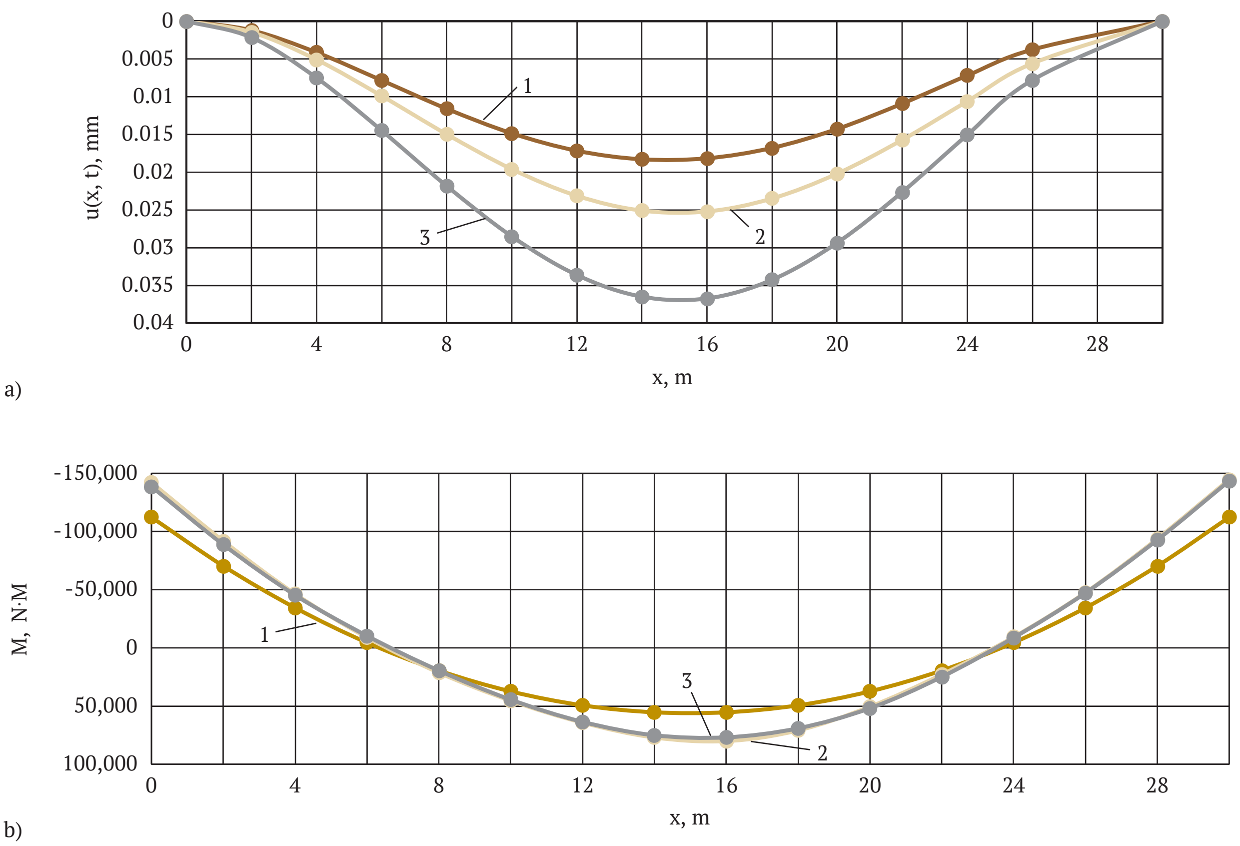


Figure 4. Displacement of the oil pipeline axis points (a) and its bending moments (b) during the sequential pumping of petrol and diesel fuel for small time values

Note: a) 1 = 0.3 s, 2 = 0.35 s, 3 = 0.45; b) 1 = 0 s, 2 = 0.15 s, 3 = 0.45 s
Source: created by the authors

Substituting q_1 and q_3 into the above formula, the bending moments of the pipeline for petrol pumping are obtained as $M_p = 112.3$ kN·m and for diesel fuel pumping as $M_d = 128.8$ kN·m. The largest bending moment during the sequential pumping of petrol and diesel fuel is 1.29 and 1.12 times higher than the bending moments during their separate pumping, respectively. When the end cross-sections of the diaphragm seal (rubber piston) enter the overhead part of the oil pipeline, which occurs at times $t = 0$ s and $t = 0.38$ s, at times $t = 0.15$ s and $t = 0.45$ s, the maximum (37 mm) deflections of the oil pipeline axis in one of its middle cross-sections in the part occur, and the end part of the pipeline section $x = 30$ m has the largest bending moments by modulus. This confirms the fact that all changes in the deflection of the pipeline axis and its bending moment in the part are caused by density oscillations between the diaphragm seal and the pumped oil products. The difference in density between the diaphragm seal and the pumped oil products leads to an increase in the deflection arc of the pipe axis at the beam crossing. This, in turn, causes a loss of pipeline stability at this crossing due to the lower axial compressive force of the pipes caused by changes in ambient temperature.

Discussion

The absence of publications found by the authors that would directly investigate the issue of sequential pumping of various oil products on the overhead part of the pipeline demonstrates the importance of current and further research in this area. However, an in-depth study of certain aspects of the topic can shed light on the technological challenges in this process, so the following studies reviewed in this chapter are worthy of attention. M.J. Brennan *et al.* (2018) investigated wave propagation due to oscillations caused by a natural gas leak from a damaged pipeline. The developed simulation model of wave propagation includes two parts: a model of amplitude damping and a model of wave propagation. It is shown that this model can be used to determine the location of a gas leak. A number of studies have focused on the behaviour of pipelines on soft ground, such as the study by J. Tang *et al.* (2023), and on permafrost, such as the study by K. Wang *et al.* (2023). The change in pipe deformations and stresses over time has been established, but these models do not take into account the oscillations of the pipes during the transportation of hydrocarbons, which reduces the informational value of the models. To improve the operating conditions of pipelines, P.O. Ayegba *et al.* (2021) used various design and technological measures but do not take into account the occurrence of pipe oscillations. As noted by Y. Zhang *et al.* (2023), the safe operation of pipelines is currently the main challenge for the pipeline industry. The results of the conducted studies show that the assessment of the vulnerability of oil and gas pipelines is at a preliminary stage and does not take into account their oscillations.

The main causes of pipeline accidents are weld defects, as described by F. Wang *et al.* (2023), and corrosion damage, which is considered by T. Arumugam *et al.* (2023). These phenomena are stimulated by pipeline oscillations during operation, which lead to the occurrence of load cycles and acceleration of corrosion processes and fatigue destruction of steels. Therefore, when designing pipelines, in particular, overhead pipeline crossings, it is necessary to take into account their oscillations to ensure high performance. The peculiarities of stress and displacement distribution in pipeline systems have been studied in many works, including those by X. Wang *et al.* (2022) and X. Li *et al.* (2022), and general approaches to such problems can be divided into three global areas. The first direction is based on analytical methods, such as the studies by M. Witek (2021) and W. Zhang *et al.* (2022). Studies in the second area, namely A. Tsatsis *et al.* (2022) and X. Liu *et al.* (2022), are based on the finite element method and use engineering software packages. The third direction – P. Ni *et al.* (2023) – used laboratory and field measurements or modelling experiments. The study presented in this article develops the first, analytical direction of research. It is worth mentioning the study of the influence of vibration parameters during pipe welding on the state of pipeline systems, as in the study by Y. Bao *et al.* (2023a), as well as the detection of defects in them caused by these oscillations, as discussed by P. Kumar & P.K. Mohapatra (2022). Mathematically, oscillatory processes are described by hyperbolic equations or their systems, whose solutions in the class of integer functions often have the property of index boundedness in the direction of oscillations (Bandura & Skaskiv, 2018) or in a set of variables (Bandura & Skaskiv, 2019).

In the publication I. Chudyk *et al.* (2020) the relationships between the parameters of unsteady longitudinal and torsional oscillations of a drilling tool can be found and a mathematical model to study their properties is developed. As a result of solving this problem, it became possible to select the optimal modes of dynamic loading of a drilling tool in order to increase its energy efficiency. This indicates the possibility of studying the oscillatory processes that accompany various oscillatory processes. In the author's problem, the alternating oscillations during the sequential pumping of two different oil products lead to an increase in bending moments, which must be taken into account when designing oil pipelines. Forced oscillations can accompany certain technological processes and are harmful to them. Their study makes it possible to obtain results that can be used to reduce the magnitude of these oscillations. This conclusion is confirmed by the work of V. Moysyshyn *et al.* (2021), in which experimental studies of the drilling process on a drilling rig made it possible to provide practical recommendations for reducing the harmful effects of drilling tool vibrations and reducing the energy intensity of the drilling process. But there are also technological processes that require specially created vibrations. In the study by Y. Bao *et al.* (2023b) certain vibrations are set and the transient characteristics of two-phase gas-liquid flow in inclined pipes are found. It was found that inclined vibration has the most significant effect on gas-liquid flow, followed by horizontal and vertical vibration. The effect of vibration on the secondary flow increases with increasing vibration amplitude. This study should be applied to offshore oil and gas equipment.

The research of D. Guan *et al.* (2019) is devoted to the experimental study of the interaction between flow fields, scour and forced vibrational pipelines in a calm water environment. In reality, a pipeline lying on the seabed is subjected to scour around it under the influence of current fields. On the other hand, the same scour occurs under the influence of quite specific vibrations of the pipeline itself, if there are no flow fields around the pipeline. This means that there is a definite connection between the scour of the channel in which the pipeline is located and the vibration of the pipeline itself. Damping oscillations at energy facilities such as pumping stations is important, as oscillations from pumping stations are added to the oscillations of oil pipelines caused by the process of pumping oil products. More and more refurbished oil pipes are being used in oil and gas transmission pipelines in China. In the publication by S. Zhang *et al.* (2023), the case of cracks in a repaired oil pipe after eight months of operation is investigated. As a result of a series of experiments, the authors concluded that during the repair work, an external weld was made at the end of the external thread of the pipe, which resulted in the formation of a martensitic metal structure, and cold cracks appeared in the zone of thermal influence, which were detected after a rather long operation as part of the pipeline. The authors do not indicate the conditions of its operation. If the pipeline was pumping oil, this process would be accompanied by pipeline oscillations. These vibrations, in turn, would stimulate the process of cracking.

Summarising all of the above in relation to the cited literature, the following should be noted: there are technological processes that generate oscillations (vibrations) of pipes in oil pipelines and other pipelines. These oscillations are harmful, and their reduction would improve the technological process and ensure the trouble-free operation of pipelines. In this case, they should be taken into account when calculating the strength and stability of pipelines. Less common are the technological processes in which certain vibrations are required to produce them. In this case, vibrations are not harmful. The authors of the study did not find any publications on the sequential pumping of two different oil products through pipelines, which shows the need to study this issue.

Conclusions

A mathematical model of the sequential pumping of two different oil products with a diaphragm seal between them through an overhead pipeline crossing has been developed to determine both the deflection function of the pipe axis at the crossing and their bending moments relative to the horizontal coordinate axis *X*. The model takes into account static forces (weights of oil products and the diaphragm seal) and the initial condition of the problem, which corresponds to the stationary state of oil products in the pipeline. The weight of pipes at the crossing, inertial forces of oil products, the diaphragm seal and oil pipeline pipes are not taken into account. As a result of the implementation of the mathematical model, it was found that from the moment one of the end cross-sections of the diaphragm seal enters the overhead crossing of the oil pipeline, which at the initial moment of time was on the border with the overhead crossing, the alternating oscillations of the oil pipeline axis begin. At the same time, smaller values of deviations from the abscissa axis *X* correspond to negative values of the axis deflection function, and larger values correspond to its positive values. The obtained function of deflections of the oil pipeline axis at the overhead crossing was used to find the function of bending moments, which, like deflections, continuously change both in modulus and sign. The entry of each of the end cross-sections of the diaphragm seal during the sequential pumping of two different oil products into the overhead part of the pipeline causes maximum positive displacements of its axis points and the largest modulus values of bending moments, which occur literally in a tenth of a second after the entry of the end section of the diaphragm seal into the overhead crossing of the pipeline. At the same time, for a 30 m long overhead crossing, the vertical displacement reaches a maximum of 37 mm from the moment of entry of the diaphragm seal, which is 0.45 s.

The maximum bending moment modulus on the overhead part of the pipeline, when two different oil products are pumped sequentially, is significantly higher than the maximum bending moment of the pipeline on the same part when each of the two oil products is pumped separately. In particular, when petrol and diesel fuel are pumped sequentially, the largest bending moment is 1.29 and 1.12 times higher than the bending moments when they are pumped separately, respectively. After both end sections of the diaphragm seal enter the overhead crossing of the oil pipeline, with an increase in the time of movement of the pumped oil products in the crossing, the displacement of the points of the oil pipeline axis in it and the bending moments of the pipes decrease. All of the above phenomena, which occur when two different oil products are pumped sequentially with a diaphragm seal between them, are caused by density jumps between the diaphragm seal and each of the pumped oil products. This study did not take into account the weight of the part of the pipeline itself on the deflection of its axis and the value of bending moments on the part. Taking this factor into account will be the goal of further research.

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Conflict of Interest

None.

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Вимушені коливання нафтопроводу на надземному переході під час послідовного перекачування різних нафтопродуктів

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Анотація. Метод послідовного перекачування різних нафтопродуктів, розділених між собою роздільником середовищ, є одним із найбільш поширених та економічно вигідних, проте, виникають значні коливання нафтопроводів на їх надземних ділянках через різницю густин. Дослідження таких коливань і їх впливу на міцність та стійкість надземних ділянок нафтопроводів практично відсутні, що робить цю тему актуальною. Метою даної статті було визначення коливань осі нафтопроводу та згинальних моментів, які водночас виникають, без врахування сил інерції перекачуваних нафтопродуктів для однопрогінного балкового переходу без компенсаторів повздовжніх деформацій. Побудовано математичну модель послідовного перекачування трубопроводом двох різних нафтопродуктів. Задача розв'язувалася методом розкладання шуканого розв'язку в ряд по власних функціях задачі вільних коливань надземної ділянки нафтопроводу зі застосуванням методу Фур'є. У результаті виконання обчислень за отриманим розв'язком задачі було встановлено, що в надземній ділянці нафтопроводу під час послідовного перекачування різних нафтопродуктів виникають знакозмінні коливання осі нафтопроводу відносно осі абсцис функції прогину осі нафтопроводу. Під час входження кінцевих перерізів роздільника в надземну ділянку нафтопроводу через десяті долі секунди в одному із середніх перерізів ділянки виникають найбільші прогини. Згинальні моменти мають найбільші значення за модулем. Криві прогину осі нафтопроводу та криві згинальних моментів для цих моментів часу практично співпадають. Встановлено, що найбільші за модулем згинальні моменти трубопроводу під час послідовного перекачування двох різних нафтопродуктів є суттєво більшими, ніж згинальні моменти цих же нафтопродуктів при їх окремому перекачуванні. Одержані результати дослідження будуть корисні на практиці проектувальникам трубопроводів, по яких послідовно перекачуватимуть нафтопродукти з різною густиною

Ключові слова: однопрогінний балковий перехід; вісь труби; власні функції задачі вільних коливань; прогини осі трубопроводу; роздільне середовище; згинальний момент